= 12.8%

σp^2 = $\sigma_A^2 W_A^2 + \sigma_B^2 W_B^2 + 2 \times W_A \times W_B \times Cov_{AB} + 2 \times W_A \times W_C \times Cov_{AC} + 2 \times W_B \times W_C \times Cov_{BC}$

= $1625 \times 0.25^2 + 2925 \times 0.40^2 + 2 \times 0.25 \times 0.40 \times 0.8 + 1.2 \times 0.25^2$

= $689.59\ (\%)^2$

= $\sqrt{689.59}$ = 26.26%

Question – 42

Mr. Abhishek is interested in investing ₹ 2,00,000 for which he is considering following three alternatives:

(i) Invest ₹ 2,00,000 in Mutual Fund X (MFX)

(ii) Invest ₹ 2,00,000 in Mutual Fund Y (MFY)

(iii) Invest ₹ 1,20,000 in Mutual Fund X (MFX) and ₹ 80,000 in Mutual Fund Y (MFY)

Average annual return earned by MFX and MFY is 15% and 14% respectively. Risk free rate of return is 10% and market rate of return is 12%.

Covariance of returns of MFX, MFY and market portfolio Mix are as follow:

	MFX	**MFY**	**Mix**
MFX	4.800	4.300	3.370
MFY	4.300	4.250	2.800
Mix	3.370	2.800	3.100

You are required to calculate:

(i) Variance of return from MFX, MFY and market return,

(ii) Portfolio return, beta, portfolio variance and portfolio standard deviation,

(iii) Expected return, systematic risk and unsystematic risk; and

(iv) Sharpe ratio, Treynor ratio and Alpha of MFX, MFY and Portfolio Mix

(SM TYK – 17)

Solution:

(i) Variance of X, Y & Market

Variance of X

(r between x & x is 1)

$$r_{x\,x} = \frac{Cov_{xx}}{\sigma_x \times \sigma_x}$$

$$1 = \frac{4.800}{\sigma_x^2}$$

$$\sigma_x^2 = 4.800$$

$$\sigma_y^2 = 4.250$$

$$\sigma_m^2 = 3.100$$

(ii) ER_p , β, σ_p^2 , & σ_p

$$ER_p = (15 \times 0.6) + (14 \times 0.4) = 14.6\%$$

Beta of Portfolio

$$\beta_x = \frac{COV_{xm}}{\sigma_m^2} = \frac{3.370}{3.100} = 1.087$$

$$\beta_y = \frac{COV_{xm}}{\sigma_m^2} = \frac{2.800}{3.100} = 0.903$$

$$\beta_p = (1.087 \times 0.6) + (0.903 \times 0.4) = 1.0134$$

$$\sigma_p^2 = \sigma_x^2 \times w_x + \sigma_y^2 \times w_y + 2 \times w_x \times w_y \times Cov_{xy}$$

$$= (4.800 \times (0.6)^2 + (4.250 \times (0.4)^2) + 2 \times 0.6 \times 0.4 \times 4.300$$

$$= 4.472\ (\%)^2$$

$$\sigma_p = 2.1147\%$$

(iii) ER as per CAPM SR & UR:

ER $= R_f + (R_m - R_f)\ \beta$

X $= 10 + (12 - 10)\ 1.087$ $= 12.17\%$

Y $= 10 + (12 - 10)\ 0.903$ $= 11.806\%$

ER_P $= 10 + (12 - 10)\ 1.0134$ $= 12.027\%$

SR $= B^2\sigma_m^2$

X $= (1.087)^2 \times 3.100$ $= 3.6628\ (\%)^2$

Y $= (0.903)^2 \times 3.100$ $= 2.5278(\%)^2$

SR_p $= (1.0134)^2 \times 3.100$ $=3.1836\ (\%)^2$

UR $= TR - SR$

X $= 4.800 - 3.6629$ $= 1.1371\ (\%)^2$

Y $= 4.250 - 2.5278$ $= 1.7222\ (\%)^2$

Z $= 4.472 - 3.18636$ $= 1.289\ (\%)^2$

(iv) (a) Sharpe Ratio $= \dfrac{ER - RF}{S.D.}$

X $= \dfrac{15 - 10}{2.19} = 2.28$

Y $= \dfrac{14 - 10}{2.06} = 1.94$

Portfolio $= \dfrac{14.6 - 10}{2.11} = 2.18$

(b) Treynor Ratio $= \dfrac{ER - R_f}{\beta}$

X $= \dfrac{15 - 10}{1.087} = 4.60$

Y $= \dfrac{14 - 10}{0.903} = 4.43$

Portfolio $= \frac{14.6 - 10}{1.0134} = 4.54$

Alpha $= ER - K_e$

X $= 15 - 12.174 = 2.826\%$

Y $= 14 - 11.806 = 2.194\%$

Portfolio $= 14.6 - 12.027 = 2.573\%$

Question – 43

The returns on stock A and market portfolio for a period of 6 years are as follows:

Year	Return on A (%)	Return on market portfolio (%)
1	12	8
2	15	12
3	11	11
4	2	-4
5	10	9.5
6	-12	-2

You are required to determine:

(i) Characteristic line for stock A

(ii) The systematic and unsystematic risk of stock A.

(SM TYK – 11)

Solution:

Characteristic line is given by

$\alpha + \beta_{Rm}$

$\beta i = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n(x)^2}$

$\alpha i = \bar{y} - \beta \bar{x}$

Return on A (Y)	Return of market (X)	xy	x^2	$(x - \bar{x})$	$(x - \bar{x})^2$	$(y - \bar{y})$	$(y - \bar{y})^2$
12	8	96	64	2.25	5.06	5.67	32.15
15	12	180	144	6.25	39.06	8.67	75.17
11	11	121	121	5.25	27.56	4.67	21.81
2	-4	-8	16	-9.75	95.06	-4.33	18.75
10	9.5	95	90.25	3.75	14.06	3.67	13.47
-12	-2	24	4	-7.75	60.06	-18.33	335.99
38	34.5	508	439.25		240.86		497.34

$\bar{y}$ = 38/6 = 6.33

$\bar{x}$ = 34.5/6 = 5.75

$$\beta = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n(x)^2} = \frac{508 - 6\,(5.75)(6.33)}{439.25 - 6(5.75)^2} = \frac{508 - 218.385}{439.25 - 198.375}$$

$$= \frac{289.615}{240.875} = 1.202$$

$\alpha = \bar{y} - \beta\,\bar{x} = 6.33 - 1.202\,(5.75) = -0.58$

Hence the characteristic line is -0.58 + 1.202 (R_m)

Total Risk of Market = σ_{m^2} $= \frac{\sum (x-\bar{x})^2}{n} = \frac{240.86}{6} = 40.14\%$

Total Risk of Stock = $\frac{497.34}{6}$ = 82.89 (%)

Systematic Risk = $\beta i^2\,\sigma_2$ = $(1.202)^2 \times 40.14 = 57.99(\%)$

Unsystematic Risk is = Total Risk – Systematic Risk

= 82.89 – 57.99 = 24.90(%)

PART III: ARBITRAGE PRICING THEORY

Question – 44

Mr. Nirmal kumar has categorized all the available stock in the market into the following types:

(i) Small cap growth stocks

(ii) Small cap value stocks

(iii) Large cap growth stocks

(iv) Large cap value stocks

Mr. Nirmal Kumar also estimated the weights of the above categories of stocks in the market index. Further, the sensitivity of returns on these categories of stocks to the three important factor are estimated to be:

Category of Stocks	Weight in the Market Index	Factor I (Beta)	Factor II (Book Price)	Factor III (Inflation)
Small cap growth	25%	0.80	1.39	1.35
Small cap value	10%	0.90	0.75	1.25
Large cap growth	50%	1.165	2.75	8.65
Large cap value	15%	0.85	2.05	6.75
Risk Premium		6.85%	-3.5%	0.65%

The rate of return on treasury bonds is 4.5%

Required:

(a) Using Arbitrage Pricing Theory, determine the expected return on the market index.

(b) Using Capital Asset Pricing Model (CAPM), determine the expected return on the market index.

(c) Mr. Nirmal Kumar wants to construct a portfolio constituting only the 'small cap value' and 'large cap growth' stocks. If the target beta for the desired portfolio is 1, determine the composition of his portfolio.

(SM TYK – 40)

Solution:

(a) ER of Market as per APT

SCG = 4.5 + (6.85 × 0.8) + (-3.5 × 1.39) + (0.65 × 1.35)

= 5.9925%

SCV = 8.8525%

LCG = 8.477%

LCV = 7.535%

ERM

= (5.9925 × 0.25) + (8.8525 × 0.10) + (8.477 ×0.50) + (7.535 × 0.15)

= 7.7525%

(b) ER as per CAPM

B_p = (0.8 × 0.25) + (0.9 × 0.10) + (1.165 × 0.50) + (0.85 × 0.15)

= 1

ER = RF + MRP B

= 4.5 + 6.85 × 1

= 11.35%

(c) Calculation of weight

$\beta = 0.9 \times W_A + 1.165 (1 - W_A)$

$1 = 0.9 W_A + 1.165 - 1.165 W_A$

$0.265 W_A = 0.165$

$W_A = 0.62$

$W_B = 1 - W_A = 1 - 0.62 = 0.38$

% of investment in small cap value = 62%

% of investment in large cap growth = 38%

Question – 45

Mr. Kapoor owns a portfolio with the following characteristics:

	Security X	Security Y	Risk Free Security
Factor 1 sensitivity	0.75	1.50	0
Factor 2 sensitivity	0.60	1.10	0
Expected Return	15%	20%	10%

It is assumed that security returns are generated by a two factor model.

(i) If Mr. Kapoor has ₹ 1,00,000 to invest and sells short ₹ 50,000 of security Y and purchases ₹ 1,50,000 of security X, what is the sensitivity of Mr. Kapoor's portfolio to the two factors?

(ii) If Mr. Kapoor borrows ₹ 1,00,000 at the risk free rate and invests the amount he borrows along with the original amount of ₹ 1,00,000 in security X and Y in the same proportion as described in part (i), what is the sensitivity of the portfolio to the two factors?

(iii) What is the expected return premium of factor 2?

(SM TYK – 15)

Solution:

(i) Beta of Portfolio

Calculation of Weights

Own fund = 10,000

Short sell (y) = 50,000

Investment in x = 1,50,000

$$W_x = \frac{1,50,000}{1,00,000} = 1.5$$

$$Wy = \frac{-50,000}{1,00,000} = -0.5$$

Factor 1

$B_P = (0.75 \times 1.5) + (1.5 \times -0.5) = 0.375$

Factor 2

$B_P = (0.60 \times 1.5) + (1.10 \times -0.5) = 0.35$

(ii) Beta of Portfolio

Own fund = ₹ 1,00,000

R_f = ₹ 1,00,000

	₹ 2,00,000

Investment in x = 2,00,000 × 1.5 = 3,00,000

Short sell in y = 2,00,000 × -0.5 = -1,00,000

$W_x = \frac{3,00,000}{1,00,000} = 3$

$W_y = \frac{-100,000}{1,00,000} = -1$

$R_f = \frac{-1,00,000}{1,00,000} = -1$

Factor 1

B_P = (0.75 × 3) + (1.5 × -1) = 0.75

Factor 2

B_P = (0.6 ×3) + (1.10 ×-1) = 0.70

(ii) Calculation of FRP_2

$5 = 0.75\ FRP_1 + 0.60\ FRP_2$ .....(1)

$10 = 1.50\ FRP_1 + 1.10\ FRP_2$ ..... (2)

Multiply equation (1) with 1.5 & equation (2) with 0.75

$7.5 = 1.125\ FRP_1 + 0.9\ FRP_2$

$7.5 = 1.125\ FRP_1 + 0.825\ FRP_2$

$0 = 0.075\ FRP_2$

$FRP_2 = \frac{0}{0.075} = 0$

Question – 46

Mr. Tamarind intends to invest in equity shares of a company the value of which depends upon various parameters as mentioned below:

Factor	Beta	Expected Value in %	Actual Value in %
GNP	1.20	7.70	7.70
Inflation	1.75	5.50	7.00
Interest rate	1.30	7.75	9.00
Stock market index	1.70	10.00	12.00
Industrial production	1.00	7.00	7.50

If the risk free rate of interest be 9.25%, how much is the return of the share under Arbitrage Pricing Theory?

(SM TYK – 38)

Solution:

Return as per Arbitrage Pricing Theory

Factor	Actual value	Expected value	Diff	Beta	Diff × B
GNP	7.70	7.70	0.00	1.20	0.00
Inflation	7.00	5.50	1.50	1.75	2.63
Int. rate	9.00	7.75	1.25	1.30	1.63
Stock Mrk index	12.00	10.00	2.00	1.70	3.40
Ind. Production	7.50	7.00	0.50	1.00	0.50
					8.16
Risk free rate in %					9.25
Return under APT					17.41

Question – 47

An investor has categorized all the available stock in the market into the following types and the estimated weights of the categories of stocks in the market index are given below. Further, the sensitivity of returns of these categories of stocks to two factors Inflation and Stock market are also given below:

Category	Weight in Market Index	Factor 1 (Inflation)			Factor 2 (Stock Market)		
		Beta 1	Expected Value in %	Actual Value in %	Beta 2	Expected Value in %	Actual Value in %
Small Cap	20%	1.20	6.70	6.70	0.80	10.00	10.50
Medium Cap	30%	1.75	4.50	6.00	0.90	7.00	8.00
Large Cap	15%	1.30	6.75	8.00	1.165	9.00	10.00
Flexi Cap	35%	1.70	7.00	6.50	0.85	8.85	9.75